RiXtrema

RISK MANAGEMENT

Shifting Correlations:
Healing the Achilles Heel of Today's Risk Models

A Rise in Correlations?

- One often hears of “rise in correlations’ or ‘correlations going to one’
- Do we observe correlations?
- Shifting correlations mean that the model is not working and a plug is needed
- What are the empirical observations that lead us to speak of shifts in correlations?
- What are the practical consequences of ignoring the issue?
- How can we address the issue: The Simplistic and The Realistic
- The role of Stress Testing

Who proposed equal/decay weighted copula?

- Rosenberg & McKibben (1973): “Ex Ante predictions of the riskiness of common stocks – or, in more general terms, predictions of probability of returns can be based on fundamental (accounting) data for the firm and also on the previous history of stock prices.”

- Impressive way to settle complicated questions, to be sure

- Key: Time and necessity of managing large portfolios has turned the original method of using recent recent history for volatility forecasting into using recent history for copula (correlation structures) estimation

- Can we use this problem to our advantage?

- “Wrong number to put into a wrong formula to get the right price”

Correlation 'Changes' in Gaussian Samples

- A common 'conditioning fallacy' is to draw some subsample of the distribution and compare those correlations to the overall correlation

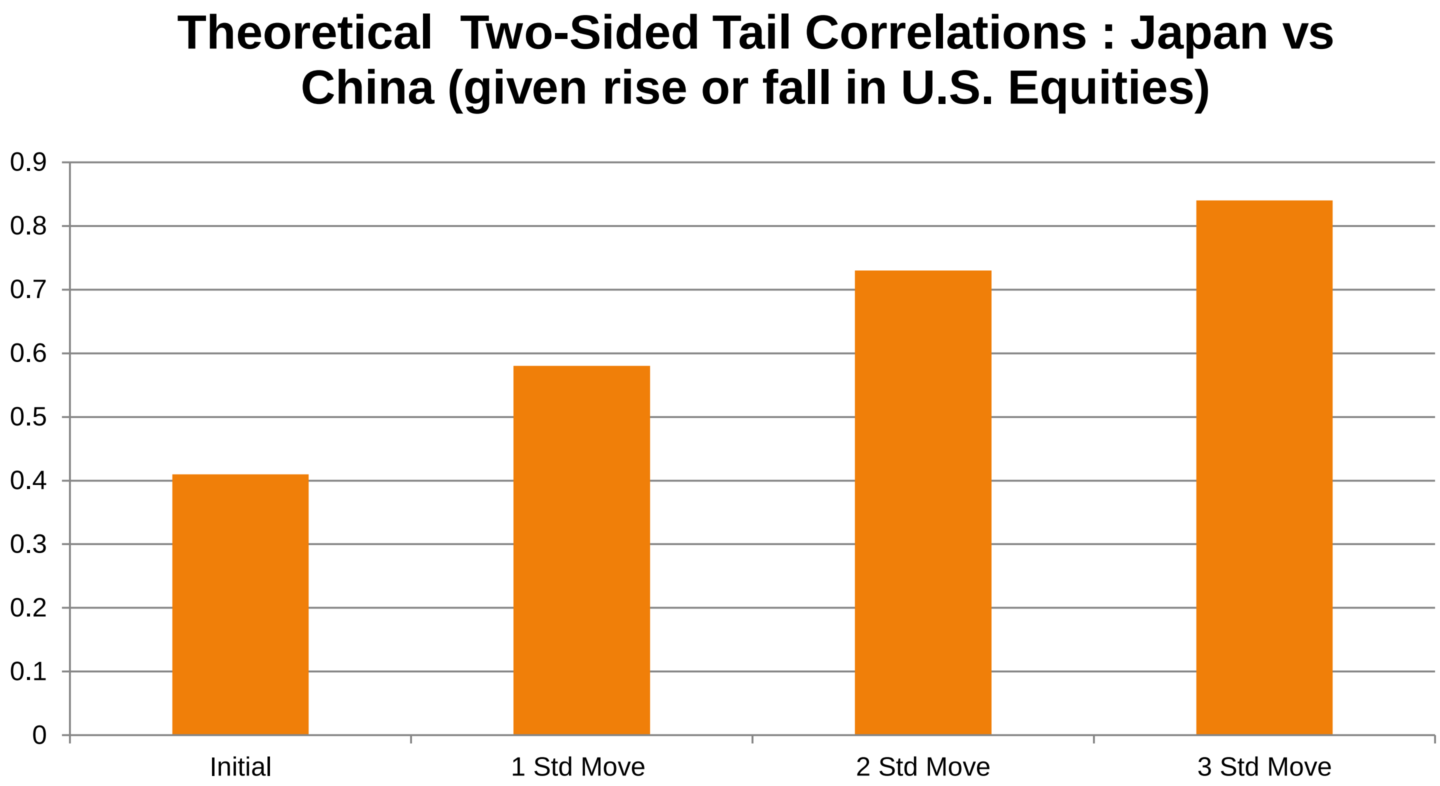
- For example, let us pick some high volatility periods and measure the correlation matrix only in those periods

- Comparing such matrix to the overall correlation combines and confuses the reality of tail correlation shifts with the theoretical properties of the Gaussian distribution

- Crucial Point: How is the conditioning conducted?

- A real difference exists between conditioning on moves in only one or both tails (correlation measured from data from both tails will always be higher, even under the perfectly normal distribution)

Beware of “Conditioning Fallacy”

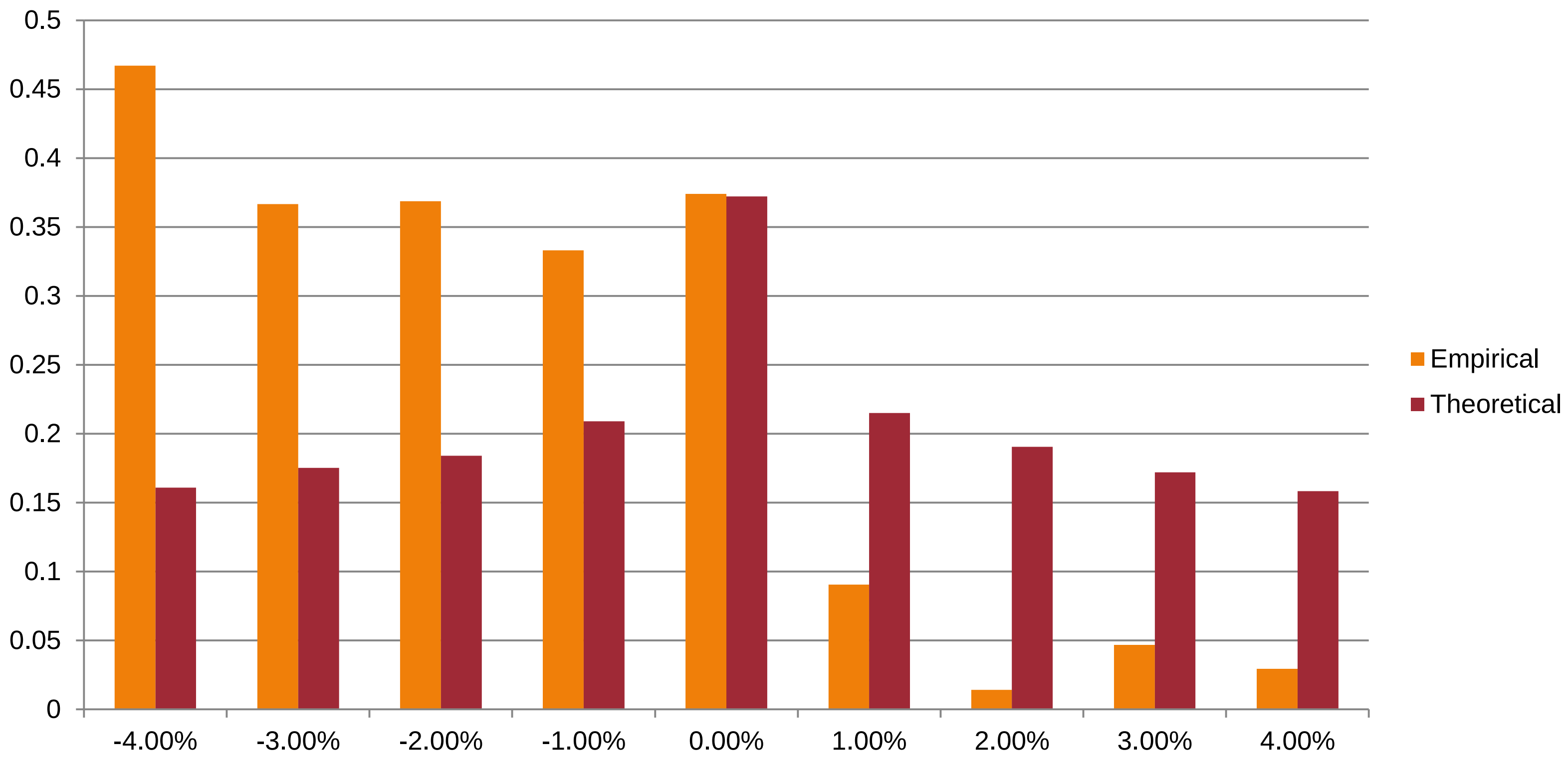


- Rising correlations in the tail under some circumstances is a <u>theoretical</u> property of the Gaussian when the observations are drawn from both tails of the distribution
- Distribution and should not be used without proper “unconditioning” statistical procedure

Obvious Solutions?

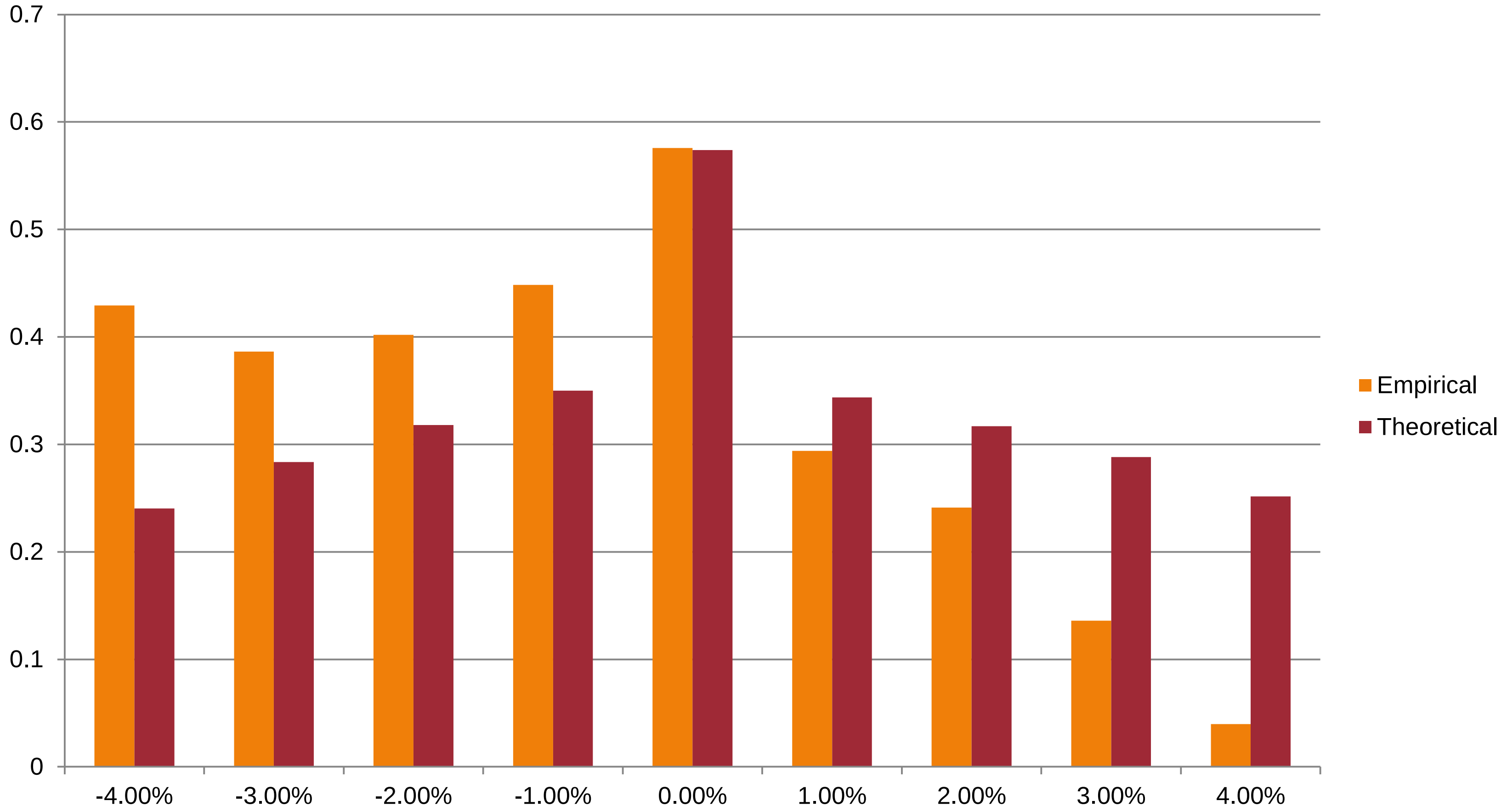
- Shocking correlations
  - – Do correlations rise linearly?
  - – Any empirical evidence behind that?
- Power Law Copula
  - – We are estimating N*(N+1)/2 parameters even for the Gaussian model
  - – Estimating power exponents would require thousands of years of financial data
- Tail Correlations
  - – Seems intuitively appealing (see the chart on the previous slide)
  - – Conditioning on a variable (volatility) closely related to the factors/assets in a model
  - – What can a poorly conditioned sample tell us?

US vs India: Did the Correlations Rise in the Left Tail?



- US vs. India equity indices (weekly returns from 1996 – 2012)
- Empirical series – correlations observed when limiting the past sample using the cutoff on the x-axis for the weekly US Equity return (i.e. -3%, picking only periods when US Equity lost more than 3%)
- Theoretical series are simulated from the covariance matrix estimated from the same data using the same cutoff condition as in the Empirical series

US vs Brazil: Is there a pattern?



- US vs Brazil equity indices (weekly returns 1996-2012)

Can Power Law Copulas Help?

- In the simplest case there is a single degrees of freedom parameter that controls the tail
- Previous slide flatly contradict this view
- Increasing the number of DoF parameters makes t-copula increasingly unstable (see Lua & Shevchenko 2010)
- Grouped t-copula (a whole other layer of uncertainty in the parameters when groups are calibrated)
- Most importantly: the GIGO Problem
- If one feeds a mix of normal and extreme periods into the copula, the result will be a hodgepodge with either Gaussian or Student

Conditional Gaussian

Given: an asset vector : $$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

possessing multivariate distribution and zero mean with covariance matrix:

$$C = \begin{pmatrix} C_{11} C_{12} \\ C_{21} C_{22} \end{pmatrix}$$

- Conditional distribution of x_2 given $x_1 = a$ has a mean

of: $$\mathrm{C}_{12} * C_{11}^{-1} * a$$

and variance of: $$C_{11} - C_{12} * C_{11}^{-1} * C_{21}$$

Tail Implied Copula

- Risk Manager Motto: “Wrong number to put into a wrong formula to get the right price”
- Traders can use Black-Scholes based on patently unrealistic GBM assumptions by making it ‘fit to reality’
- In Black-Scholes, the right price is the market price of the derivative, the wrong formula is Black-Scholes itself, and the wrong number to be entered to get the right price is implied volatility
- Tail-Implied Copula does the same for the multivariate Gaussian
- In the risk forecasting world the ‘right price’ is the diversification that is available in the tail i.e. the tail correlations that are actually observed, the wrong formula is the multivariate Gaussian and the wrong number to be entered to get the ‘right price’ is the tail-implied correlations obtained by Bayesian estimates

Tail Copula Details: Using Bayesian Intuition

Let's call the extreme matrix derived from negative observations below the bound C_{BELOW} , which is the covariance matrix of the distribution $(X_1, X_2 \mid X_1 < a)$

Let's call the extreme matrix derived from negative observations above the same bound C_{ABOVE} , which is the covariance matrix of the distribution $(X_1, X_2 \mid X_1 > a)$

$$C_{BELOW} \approx C \; - C_{A,11,12} C_{A,11}^{-1} C_{A,11,21}$$

$C_{A,11}^{-1}$ - is the inverse variance of the conditioning (shocked) variable X_1 corresponding to the distribution $(X_1, X_2 \mid X_1 > a)$

$$C_{IMPLIED} \approx C_{BELOW} + C_{A,11,12} C_{A,11}^{-1} C_{A,11,21}$$

Testing the Stress Testing: 500 Random Portfolios per Stress Test

Shock Factor	Shock Size	Start Period	End Period	RMSE (RiXtrema)	RMSE (MPT)
Diversified Fin	-43.80%	9/3/2008	11/19/2008	0.007	0.034
Italy	-24%	7/6/2011	9/14/2011	0.009	0.058
US Equity	-16%	7/14/1998	10/6/1998	.0156	.021

- Set the stress test shock equal to “Shock Size” for the “Shock Factor” on the “Start Period” date
- This shock is equal to the one that subsequently will be observed
- Use tail-implied correlations and time decay weighed variances to estimate impacts on 500 randomly constructed portfolios
- Compare the impacts to the actual returns for 500 random portfolios between “Start Period” and “End Period”
- Calculate Root Mean Squared Error (RMSE) across all portfolios

References

- Rosenberg, B. and McKibben W., 1973, “Prediction of Systemic and Specific Risk in Common Stocks”, Journal of Financial and Quantitative Analysis, vol.8, issue 02, 317:333.
- Luo, X. & Shevchenko, P., “The t-copulas with multiple degrees of freedom”, Quantitative Finance. Volume 10, Issue 9. November 2010. 1039-1054

New Paradigm of Risk Management

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